Real-Time Hybrid Simulation of a Reinforced Concrete Building using Force-Based Elements and Advanced Explicit Integration Algorithms

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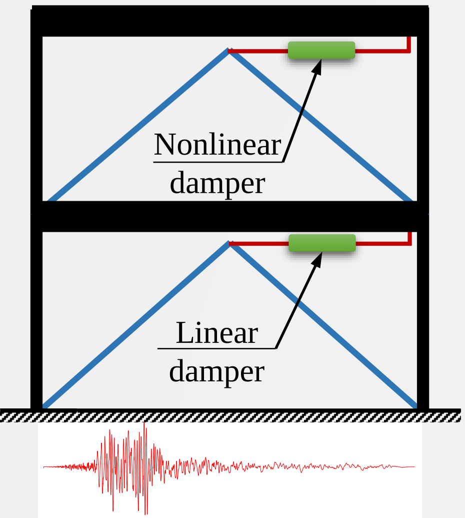
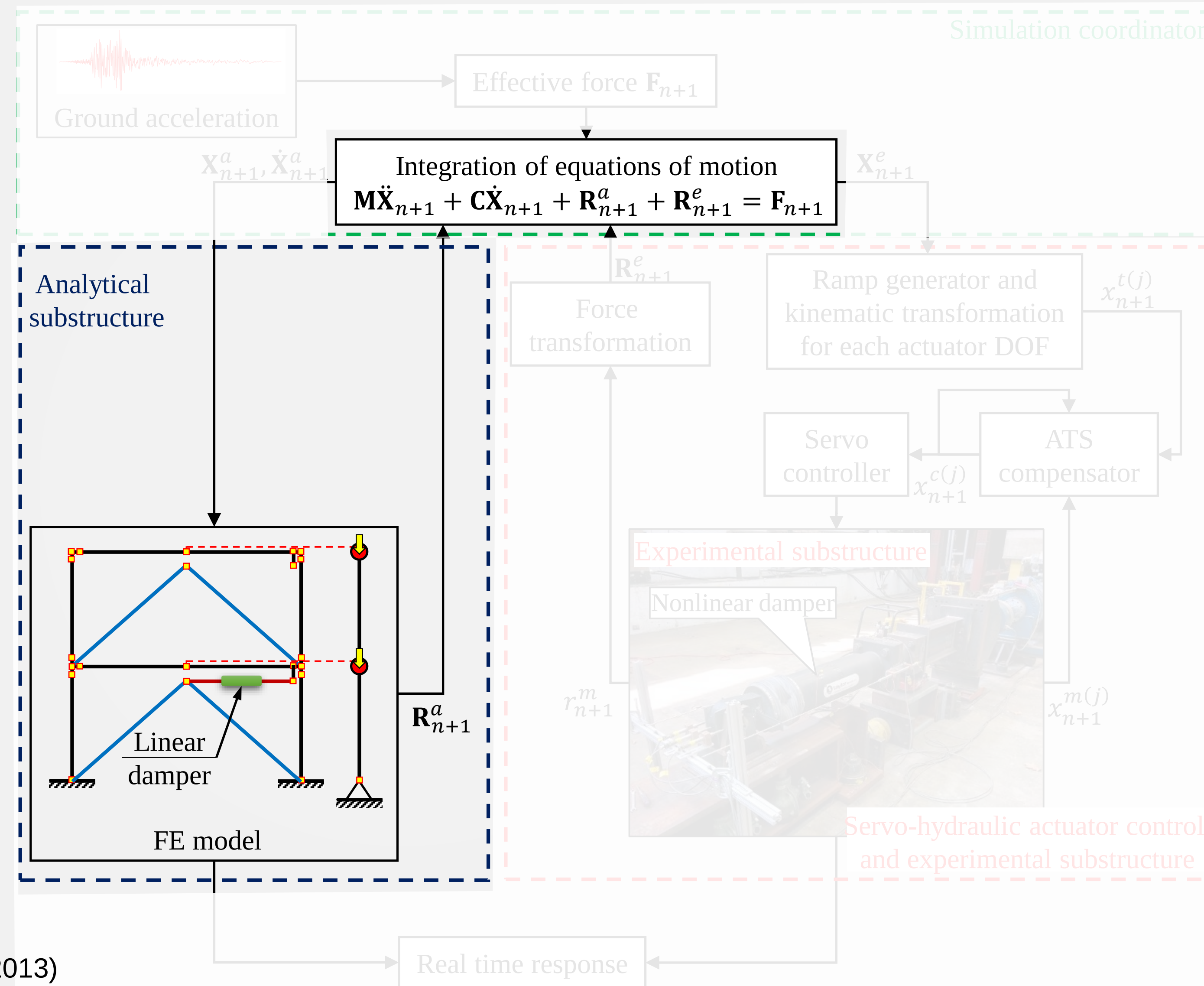
Outline

- Introduction
- Advanced explicit direct integration algorithms with numerical damping
  - Formulation
  - Numerical characteristics
- Force-based fiber element implementation
- Prototype structure
- Numerical assessment of element implementation scheme
- Real-time hybrid simulation (RTHS)
  - Model-based integration parameters
  - Stability, accuracy, and numerical dissipation
  - Influence of fixed number of element iterations
- Summary and conclusions

Introduction: RTHS





- ✓ Integration algorithm
- ✓ FE modeling

ATS Compensator (Chae et al., 2013)

Kolay, C. "Parametrically Dissipative Explicit Direct Integration Algorithms for Computational and Experimental Structural Dynamics". Ph.D. Dissertation. Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, USA, 2016

Introduction: RTHS

Direct Integration Algorithms

- Explicit formulation
- Unconditional stability
- Controllable numerical damping
- Improved overshoot for high-frequency modes
- Improved stability for nonlinear stiffening type systems

KR-α method

Modified KR-α method

Force-based fiber element implementation with fixed number of iterations

FE Modeling of Analytical Substructure

Displacement-based fiber elements

- Curvature varies linearly
  - Requires many elements per structural member to model nonlinear response
  - Increases number of DOFs
- State determination is straight forward

Force-based fiber elements

- Equilibrium is strictly enforced
  - ✓ Material nonlinearity can be modeled using a single element per structural member
  - ✓ Reduces number of DOFs
- Requires iterations at the element level

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Explicit Modified KR-α (MKR-α) Method

$\boldsymbol{\alpha_1}$, $\boldsymbol{\alpha_2}$, and $\boldsymbol{\alpha_3}$: model-based integration parameters

Velocity update: $$\dot{\mathbf{X}}_{n+1} = \dot{\mathbf{X}}_n + \Delta t \boldsymbol{\alpha}_1 \ddot{\mathbf{X}}_n$$

Displacement update: $$\mathbf{X}_{n+1} = \mathbf{X}_n + \Delta t \dot{\mathbf{X}}_n + \Delta t^2 \boldsymbol{\alpha}_2 \ddot{\mathbf{X}}_n$$

Weighted equations of motion: $$\mathbf{M}\hat{\ddot{\mathbf{X}}}_{n+1} + \mathbf{C}\dot{\mathbf{X}}_{n+1-\alpha_f} + \mathbf{K}\mathbf{X}_{n+1-\alpha_f} = \mathbf{F}_{n+1-\alpha_f}$$

where,

$$\hat{\ddot{\mathbf{X}}}_{n+1} = (\mathbf{I} - \boldsymbol{\alpha}_3)\ddot{\mathbf{X}}_{n+1} + \boldsymbol{\alpha}_3 \ddot{\mathbf{X}}_n$$

$$\dot{\mathbf{X}}_{n+1-\alpha_f} = (1-\alpha_f)\dot{\mathbf{X}}_{n+1} + \alpha_f \dot{\mathbf{X}}_n$$

$$\mathbf{X}_{n+1-\alpha_f} = (1-\alpha_f)\mathbf{X}_{n+1} + \alpha_f \mathbf{X}_n$$

$$\mathbf{F}_{n+1-\alpha_f} = (1-\alpha_f)\mathbf{F}_{n+1} + \alpha_f \mathbf{F}_n$$

Initial acceleration: $$\mathbf{M}\ddot{\mathbf{X}}_0 = [\mathbf{F}_0 - \mathbf{C}\dot{\mathbf{X}}_0 - \mathbf{K}\mathbf{X}_0]$$

Kolay, C., & Ricles, J. M. (2014). Development of a family of unconditionally stable explicit direct integration algorithms with controllable numerical energy dissipation. *Earthquake Engineering and Structural Dynamics*, *43*(9), 1361–1380. http://doi.org/10.1002/eqe.2401

Integration Parameters

- Parameter controlling numerical energy dissipation
  - ρ_∞ = spectral radius when $\Omega = \omega\Delta t \to \infty$
    - varies in the range $0 \leq \rho_\infty \leq 1$
  - ρ_∞ = 1: No numerical energy dissipation
  - ρ_∞ = 0: Asymptotic annihilation
- Scalar int
  - $\alpha_m = -$ ... $\beta = \frac{1}{2}\left(\frac{1}{2} + \gamma\right)$

MKR-α: One parameter (ρ_∞) family of algorithms

- Model-based integration parameter matrices:
  - $\boldsymbol{\alpha_1} = [\mathbf{M}_{IP} + \gamma\Delta t\mathbf{C}_{IP} + \beta\Delta t^2\mathbf{K}_{IP}]^{-1}\mathbf{M}_{IP}$; $\quad \boldsymbol{\alpha_2} = \left(\frac{1}{2} + \gamma\right)\boldsymbol{\alpha_1}$
  - $\boldsymbol{\alpha_3} = [\mathbf{M}_{IP} + \gamma\Delta t\mathbf{C}_{IP} + \beta\Delta t^2\mathbf{K}_{IP}]^{-1}\left[\alpha_m\mathbf{M}_{IP} + \alpha_f\gamma\Delta t\mathbf{C}_{IP} + \alpha_f\beta\Delta \mathrm{t}^2\mathbf{K}_{IP}\right]$
  - IP stands for integration parameters
  - $\mathbf{M}_{IP}$, $\mathbf{C}_{IP}$, and $\mathbf{K}_{IP}$ need to be formed based on the hybrid system

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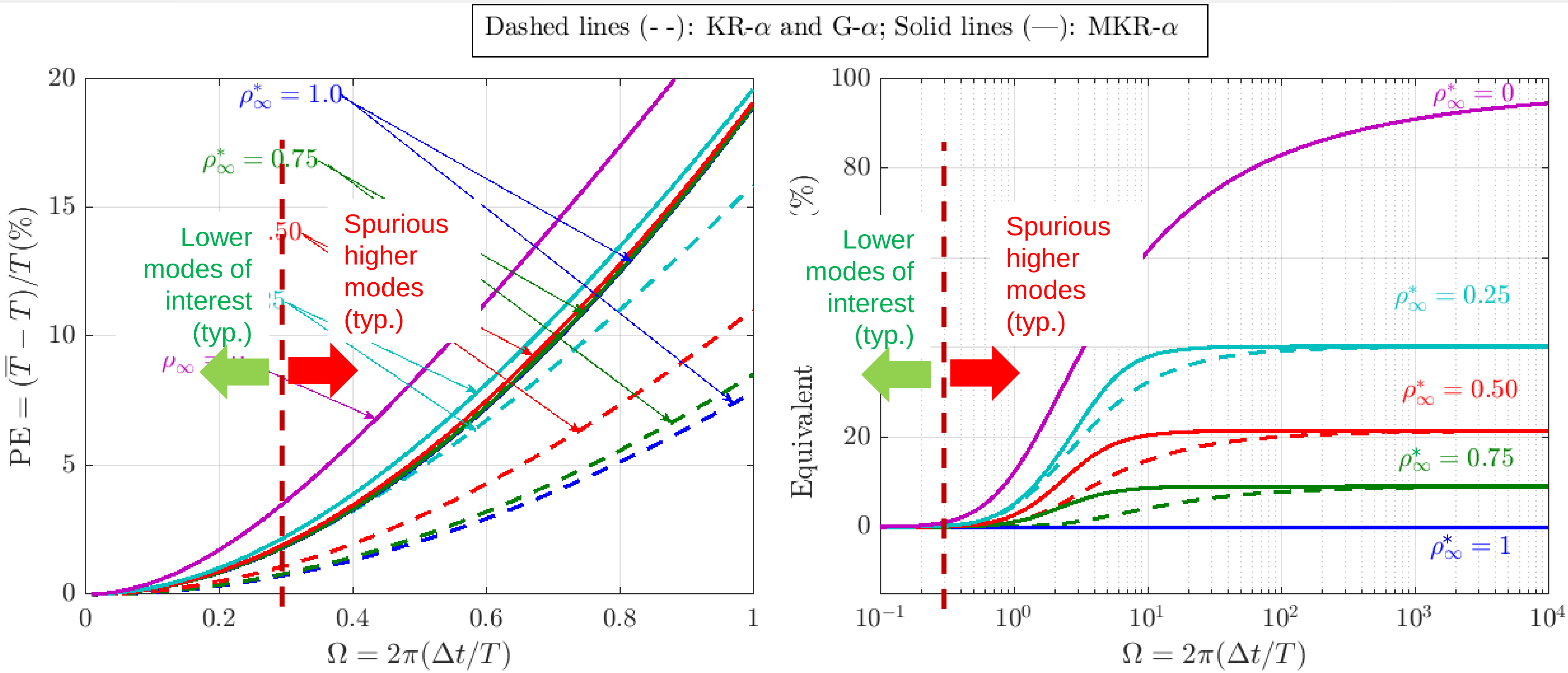
Numerical Characteristics

Compare based on same high-frequency dissipation

$$\rho_\infty^* = \begin{cases} \rho_\infty & \text{for KR}-\alpha \text{ and G}-\alpha \text{ methods} \\ \rho_\infty^2 & \text{for MKR}-\alpha \text{ method} \end{cases}$$

G-α: Implicit generalized-α method (Chung & Hulbert, 1993)



Δt = integration time step size; T = undamped natural period of an SDOF oscillator

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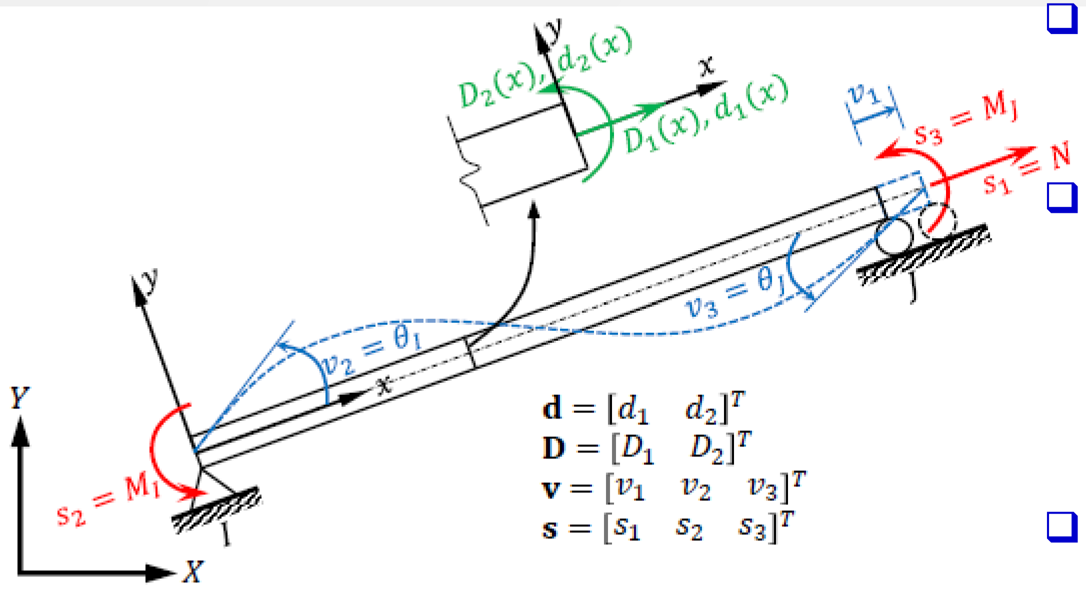
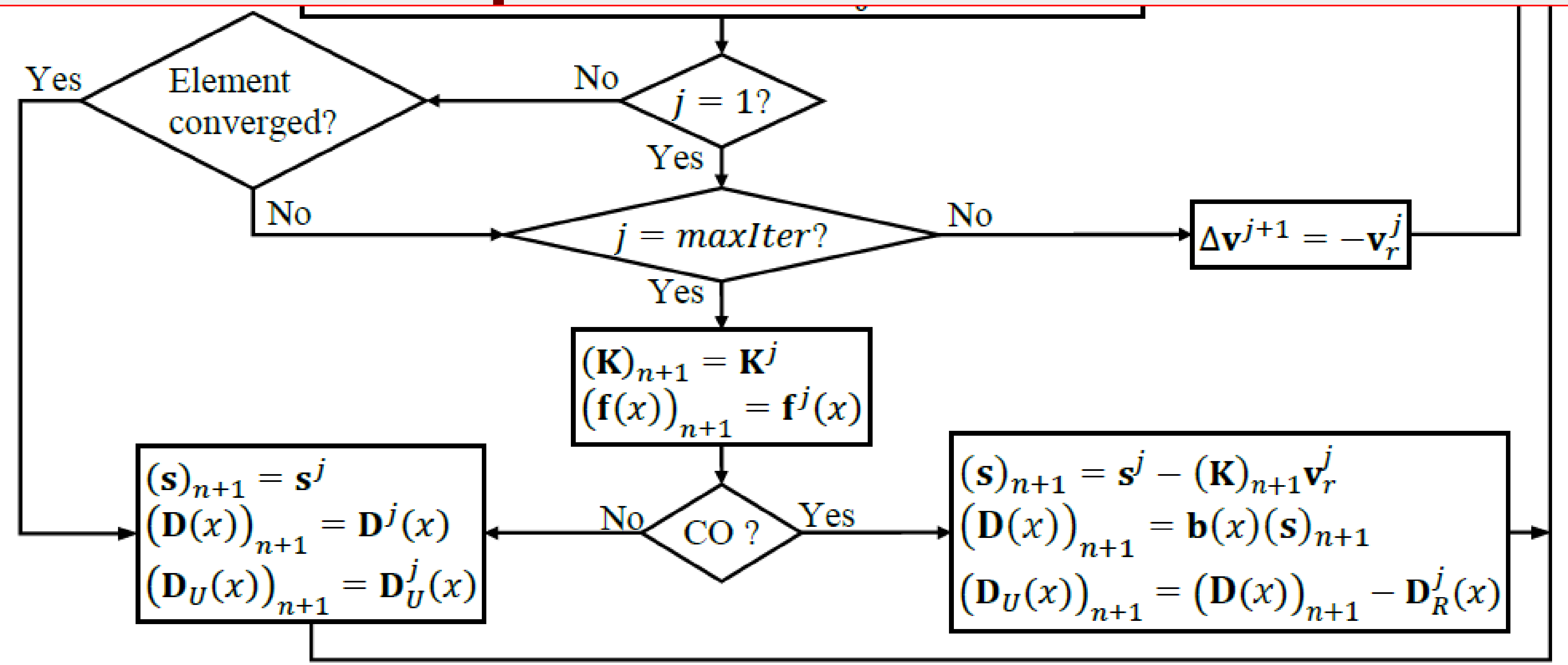
Force-Based (FB) Element State Determination

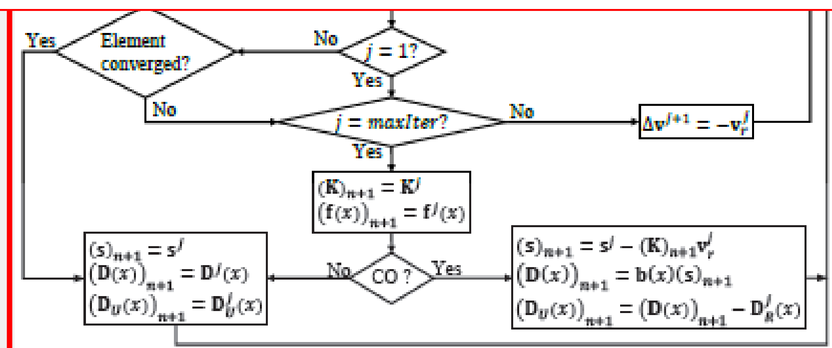
- ❑ Given element deformations $\mathbf{v}$, need element restoring forces $\mathbf{s}$
- ❑ Know the force interpolation function
  - ➢ Constant axial force and linear bending moment if no element loads
- ❑ State determination is not straight forward in a standard stiffness based FE program



- ❑ Spacone et al. (1996) developed an iterative procedure
  - ➢ Not well suited for RTHS
- ❑ Neuenhofer and Filippou (1997) proposed a noniterative procedure
  - ➢ Uses iteration at the structure level (Newton-Raphson type)
  - ➢ Not applicable for RTHS using explicit algorithms
- ❑ New implementation scheme based on Spacone et al. (1996) and Neuenhofer and Filippou (1997)
  - ➢ Fixed number of iterations
  - ➢ Carry over unbalanced section forces and correct in the next time step

FB Implementation Scheme





j = iteration index

CO = Carry over

Kolay, C., & Ricles, J. M. (2016). Force-based frame element implementation for real-time hybrid simulation using explicit direct integration algorithms. Submitted to *Journal of Structural Engineering*.

Element Convergence Criteria

- Employed the energy based criteria (Taucer et al., 1991)

$$(NEI^j)_{n+1} = \frac{(EI^j)_{n+1}}{(EI^{j=1})_{n+1}} \leq Etol \quad \text{for } j > 1$$

where

$$(EI^j)_{n+1} = (\Delta \mathbf{s}^j)^T (\Delta \mathbf{v}^j) = (\Delta \mathbf{v}_r^{j-1})^T \mathbf{K}^{j-1} (\Delta \mathbf{v}_r^{j-1})$$

$$(EI^{j=1})_{n+1} = (\Delta \mathbf{s}^{j=1})^T (\Delta \mathbf{v}^{j=1}) = ((\Delta \mathbf{v})_{n+1})^T (\mathbf{K})_n (\Delta \mathbf{v})_{n+1}$$

- A typical value of $Etol = 10^{-16}$ is used (Taucer et al., 1991)

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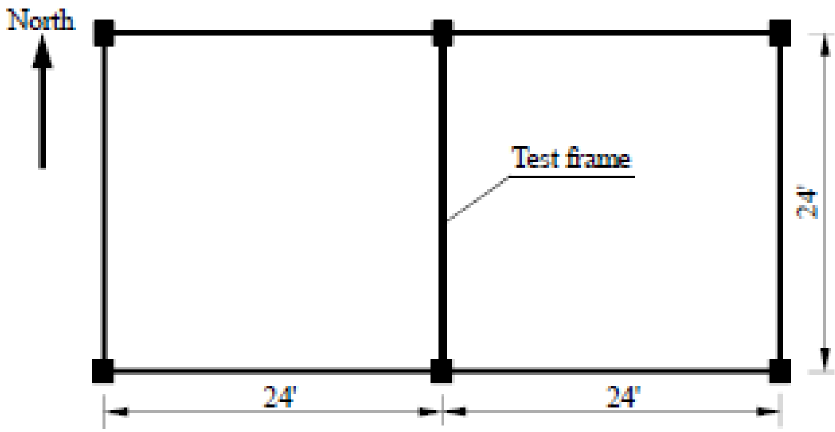
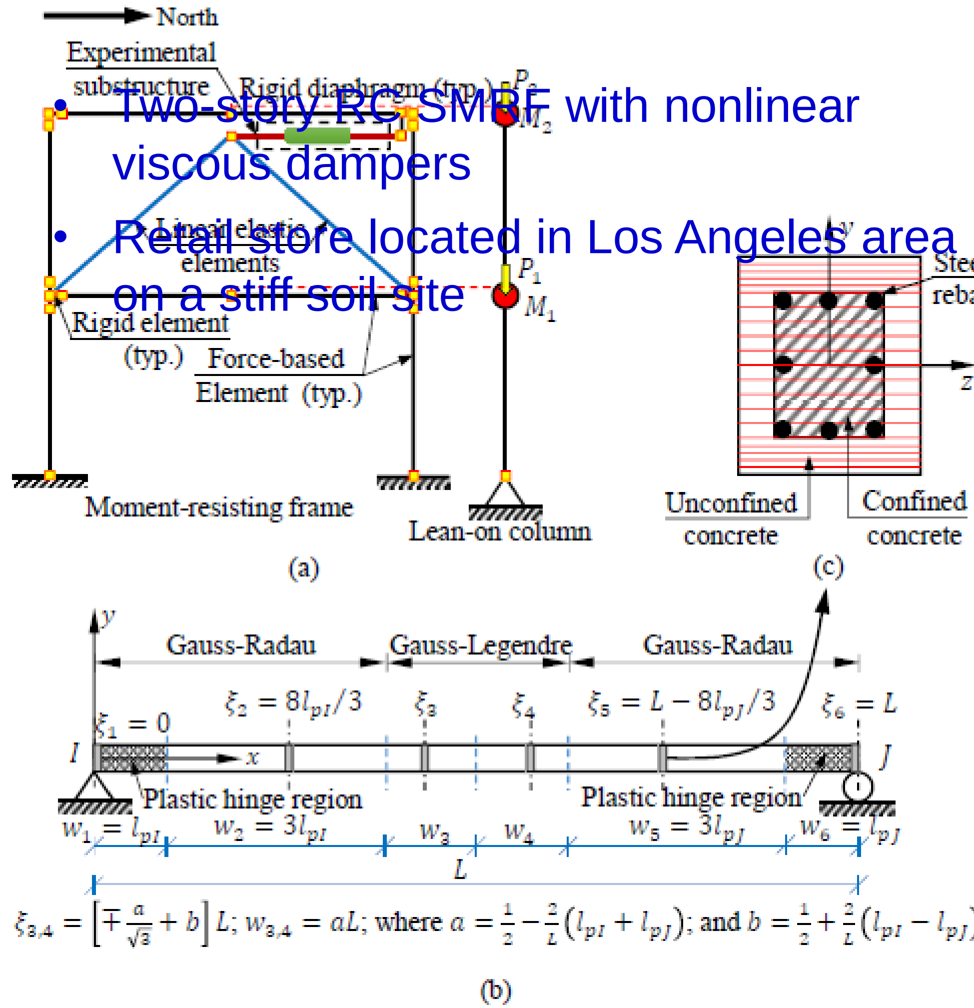
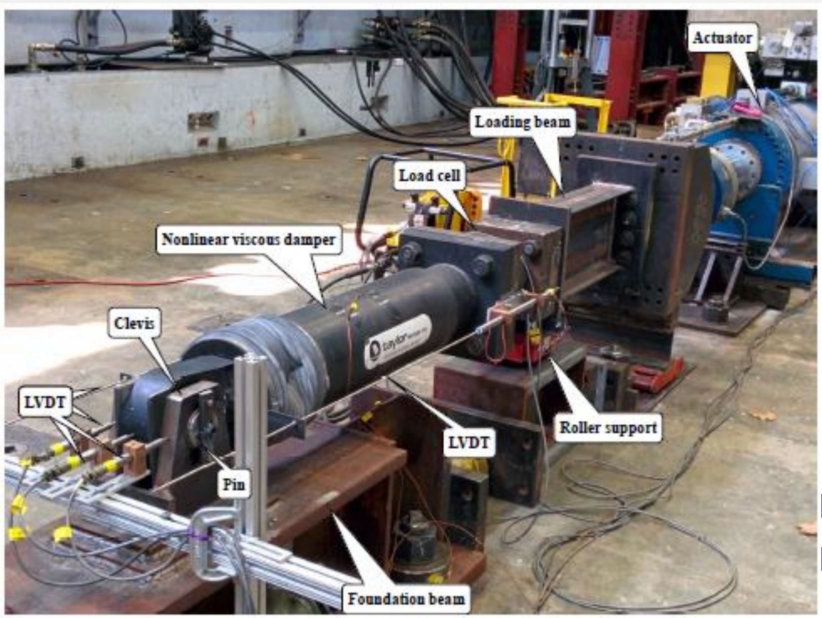
Prototype and RTHS Configuration



Prototype floor plan

- Two-story RC SMRF with nonlinear viscous dampers
- Retail store located in Los Angeles area on a stiff soil site





- ❑ 1999 Chi-Chi EQ record scaled to MCE hazard level
- ❑ Time step: $\Delta t = \frac{3}{1024}$ s

Plastic hinge integration (Scott and Fenves, 2006)

Modeling of Inherent Damping

- In RTHS using explicit algorithms generally mass and initial stiffness proportional damping is used
  - Known to produce unrealistically large damping forces when structure undergoes significant inelastic deformations
  - Can use nonproportional damping (Kolay et al., 2015)
    - Not a good model for FB elements because deformations localize at some integration points not in an entire element
- Use tangent stiffness for FB elements; it is readily available
  - For other elements, if any, use initial stiffness
  - Damping forces are calculated for each FB element inside state determination process
  - 3% damping to first ($T_1 = 0.43$ s) and second modes ($T_2 = 0.12$ s) of system

Kolay, C., Ricles, J. M., Marullo, T. M., Mahvashmohammadi, A., & Sause, R. (2015). Implementation and application of the unconditionally stable explicit parametrically dissipative KR-α method for real-time hybrid simulation. *Earthquake Engineering & Structural Dynamics*, *44*(5), 735–755. http://doi.org/10.1002/eqe.2484

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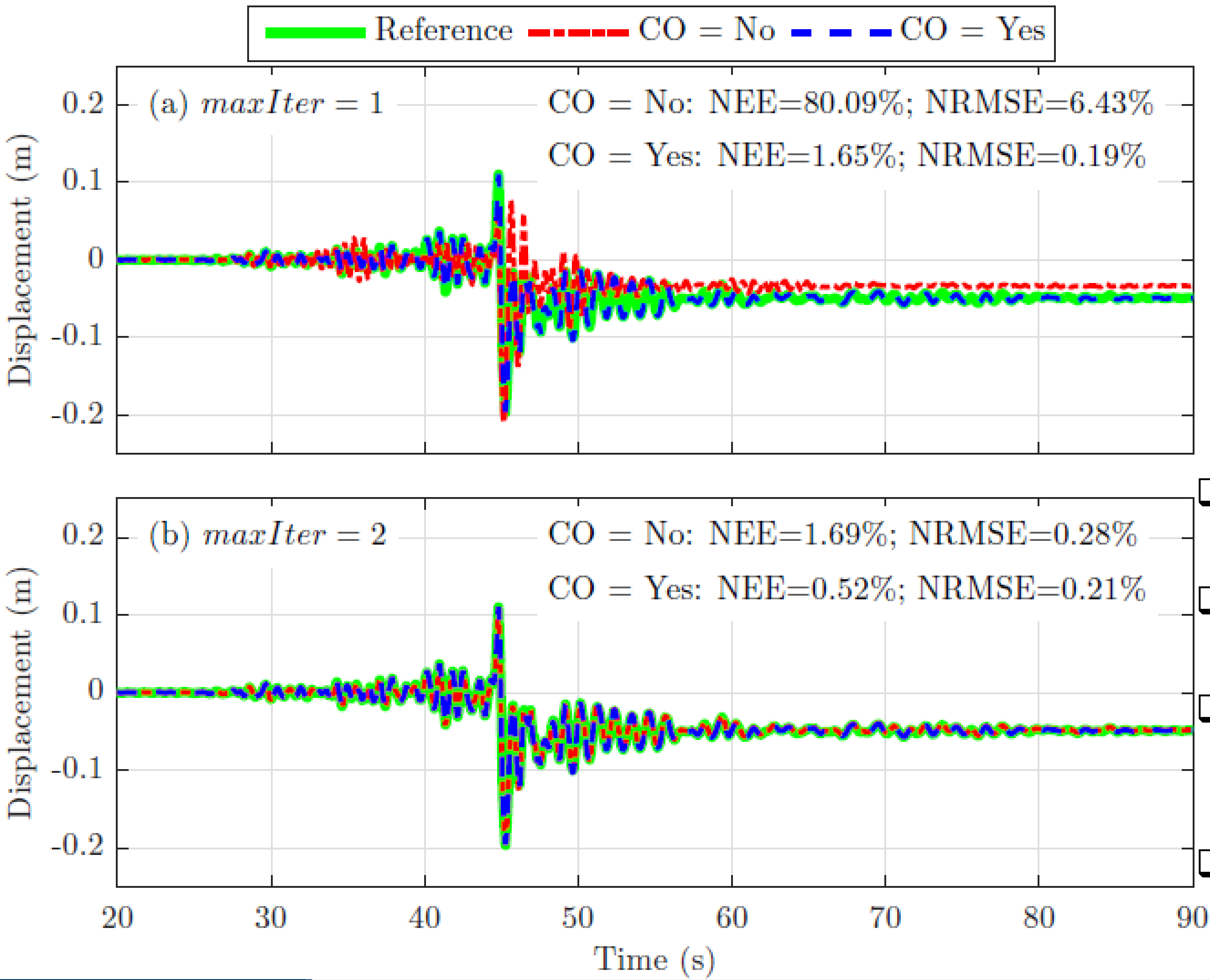
Assessment of FB Element Implementation

- Consider only the RC SMRF
- Perform numerical simulation using the same ground motion
- Study the influence of max number of iterations $(maxIter)$ with CO=Yes and CO= No based on a comparison with a reference solution
  - Reference solution: Newmark average acceleration algorithm and all the FB elements were allowed to converge with $Etol = 10^{-16}$
- Numerical damping is not required: $\rho_{\infty}^{*} = 1.0$
- Time step $\Delta t = \frac{3}{1024}$ s, smallest time step that can be used in real-time for the RTHS configuration with $maxIter = 2$ for all FB elements

Assessment of FB Element Implementation

Roof displacement from numerical simulation



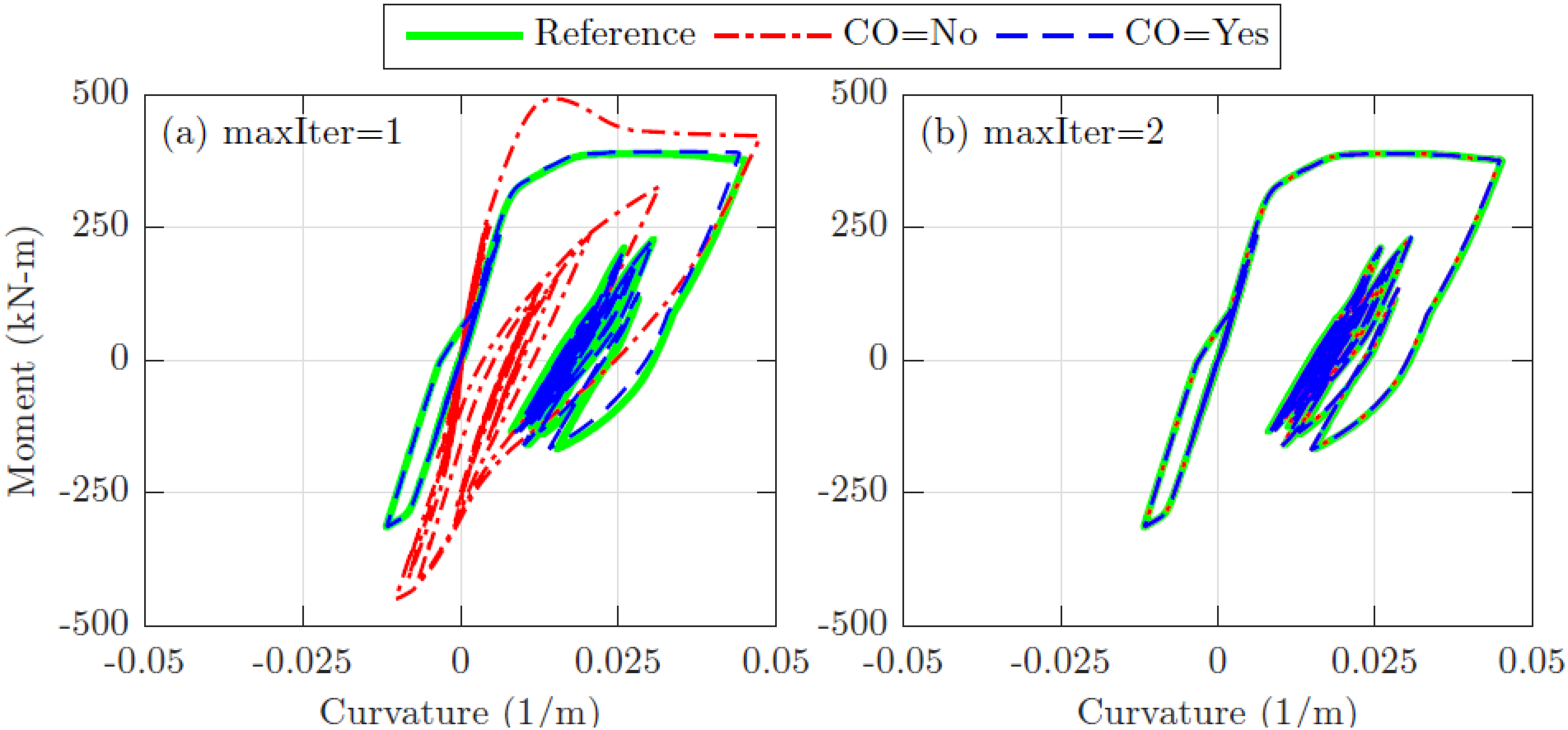
$$\mathrm{NEE} = \left| \frac{\sum_{n=1}^{N} (x_n)^2 - \sum_{n=1}^{N} (\hat{x}_n)^2}{\sum_{n=1}^{N} (\hat{x}_n)^2} \right|$$

$$\mathrm{NRMSE} = \frac{\sqrt{\frac{1}{N}\sum_{n=1}^{N} (x_n - \hat{x}_n)^2}}{\max(\hat{\mathbf{x}}) - \min(\hat{\mathbf{x}})}$$

- CO=No with $maxIter = 1$ produces large error
- CO=Yes compare well with reference
- Even $maxIter = 1$ with CO produces acceptable results
- Increasing $maxIter$ increases accuracy

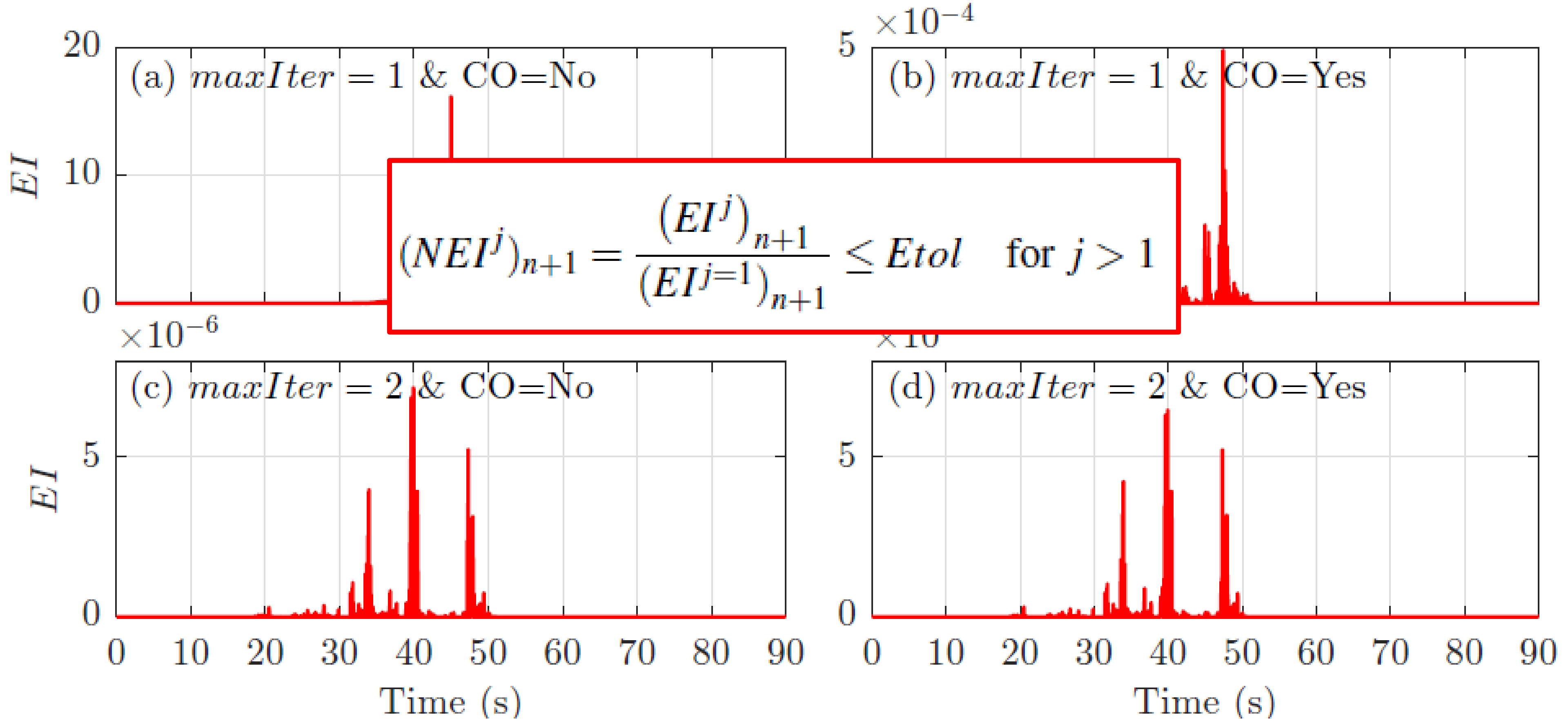
Assessment of FB Element Implementation

Moment-curvature response from numerical simulation at the first-story south side column base



Assessment of FB Element Implementation

Energy increment $\left(EI^{j=maxIter+1}\right)_n = \left(\Delta \mathbf{s}^j\right)^T \left(\Delta \mathbf{v}^j\right)$ for first-story south side column element from numerical simulation



$$(NEI^j)_{n+1} = \frac{(EI^j)_{n+1}}{(EI^{j=1})_{n+1}} \leq Etol \quad \text{for } j > 1$$

Assessment of FB Element Implementation

Peak story-drift (%) from numerical simulations with CO=Yes

Story	Reference	$maxIter = 1$	$maxIter = 2$
1	2.566	2.536	2.547
2	2.925	2.913	2.906

Assessment of FB Element Implementation

- CO=Yes produces an accurate result even if no iteration is performed at the element level ($maxIter = 1$)
- Benefit of CO=Yes reduces with increasing $maxIter$
- It is useful to perform the carry over (CO=Yes) because additional computation effort is small
  - Use only CO=Yes for RTHS

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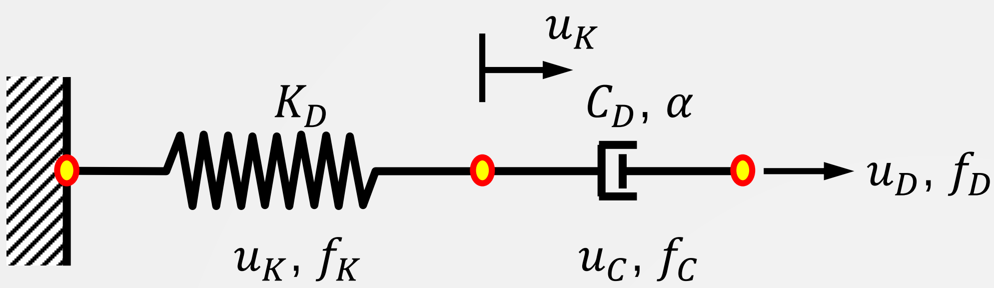
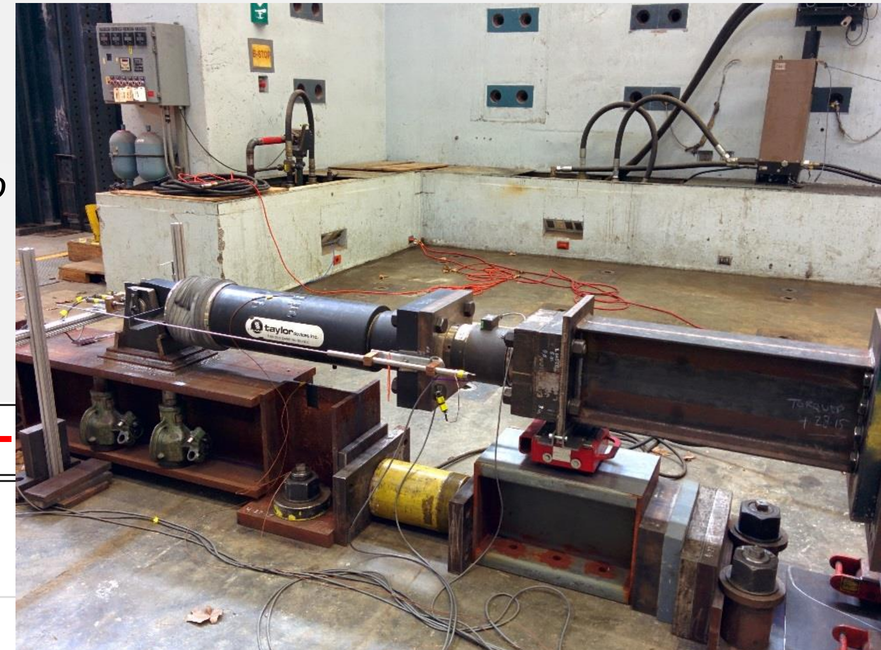
RTHS: Model-Based Integration Parameters

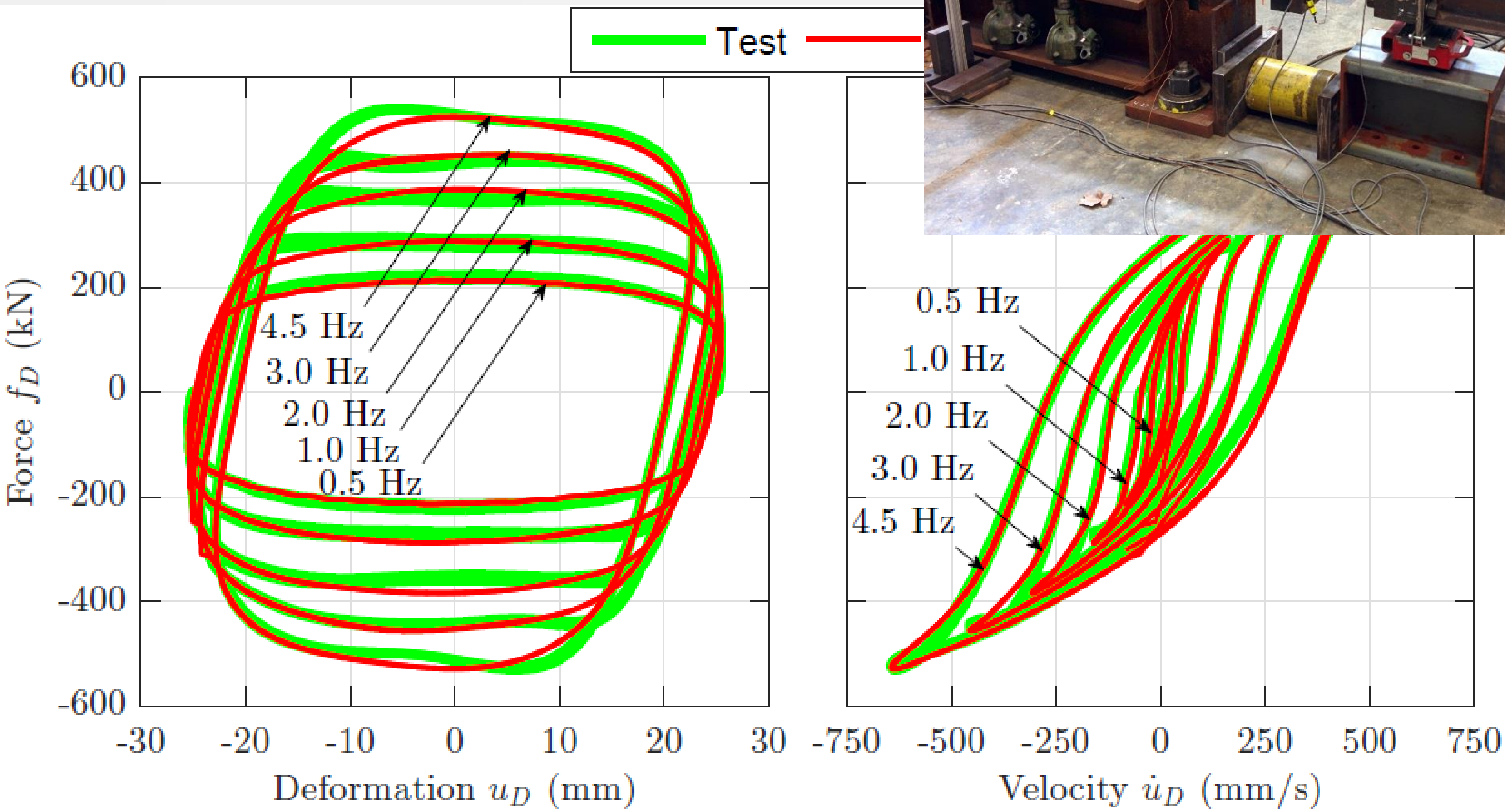
- Model-based integration parameters ($\boldsymbol{\alpha}_1$, $\boldsymbol{\alpha}_2$, and $\boldsymbol{\alpha}_3$) require $\mathbf{M}_{IP}$, $\mathbf{C}_{IP}$, and $\mathbf{K}_{IP}$
- For the present study
  - $\mathbf{M}_{IP} = \mathbf{M}$ =analytically modeled mass matrix
    - Experimental substructure mass is small
  - $\mathbf{C}_{IP} = (a_0\mathbf{M} + a_1\mathbf{K}_I^a) + \mathbf{C}_{eq}^e$
    - $\mathbf{K}_I^a$ = initial stiffness matrix of analytical substructure
    - $\mathbf{C}_{eq}^a$ =equivalent damping matrix of experimental substructure
    - a_0 and a_1 are Rayleigh damping coefficients
  - $\mathbf{K}_{IP} = \mathbf{K}_I^a + \mathbf{K}_{eq}^e$
    - $\mathbf{K}_{eq}^e$ =equivalent stiffness matrix of experimental substructure
- How can we determine $\mathbf{C}_{eq}^e$ and $\mathbf{K}_{eq}^e$?

Damper Characterization



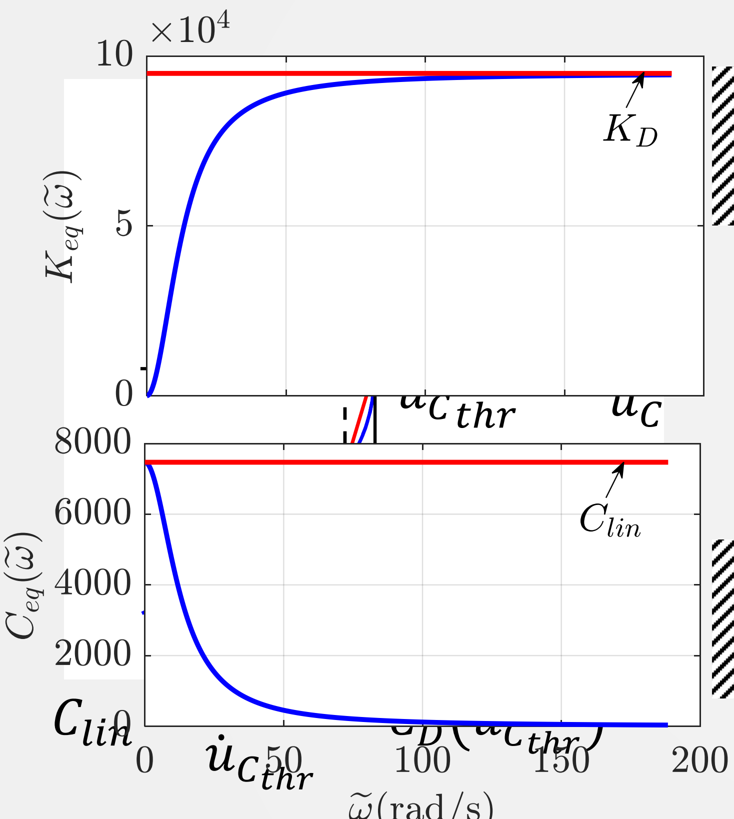
Nonlinear Maxwell damper model



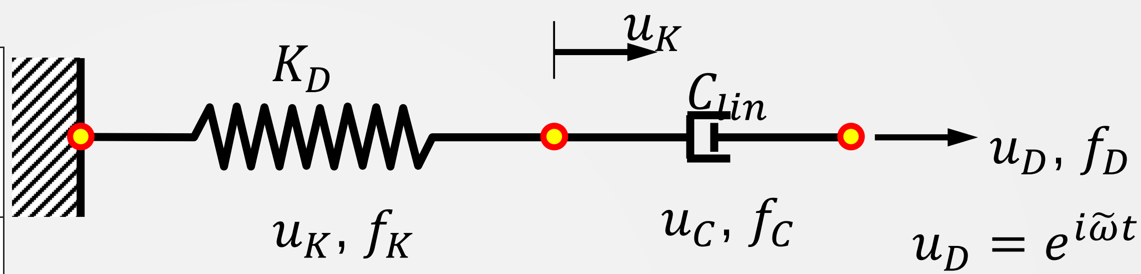


Model-Based Integration Parameters

Linearization of nonlinear Maxwell model at a small velocity (0.5 in/s) and determination of frequency dependent equivalent Kelvin-Voigt model parameters

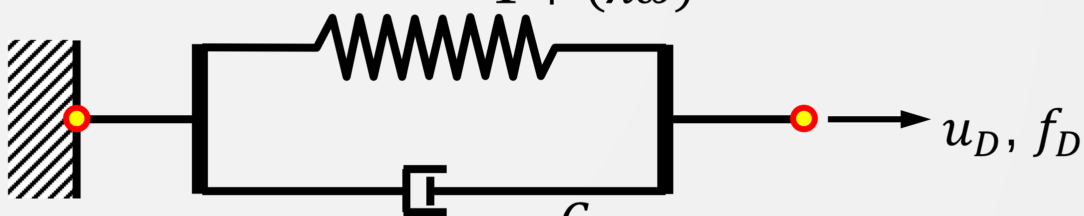


Linearization



Linearized Maxwell model

$$K_{eq}(\widetilde{\omega}) = \frac{K_D(\lambda\widetilde{\omega})^2}{1 + (\lambda\widetilde{\omega})^2}$$



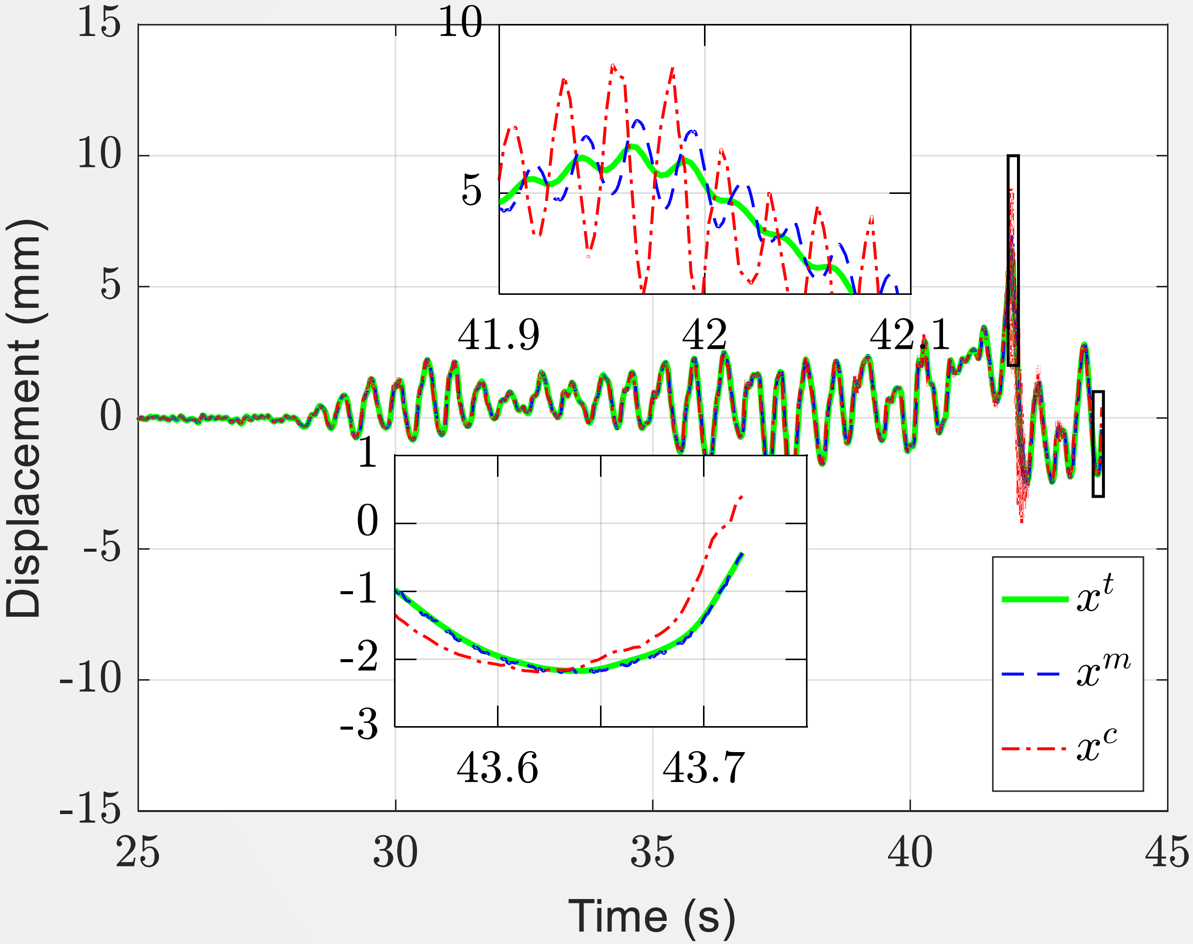
$$C_{eq}(\widetilde{\omega}) = \frac{C_{lin}}{1 + (\lambda\widetilde{\omega})^2}$$

Equivalent Kelvin-Voigt model

- ❑ What is the value of $\widetilde{\omega}$?

RTHS Results: Instability!

$\rho_\infty^* = 0.50$, $\tilde{\omega} = \omega_1$ and $maxIter = 2$ for all FB elements



High-frequency oscillations:

- Causes:
  - Underestimation error in C_{eq} & K_{eq}
  - Noise in restoring forces
  - ATS compensator amplifying higher frequencies
- Remedies:
  - Add more numerical damping
  - Increase C_{eq} and K_{eq}

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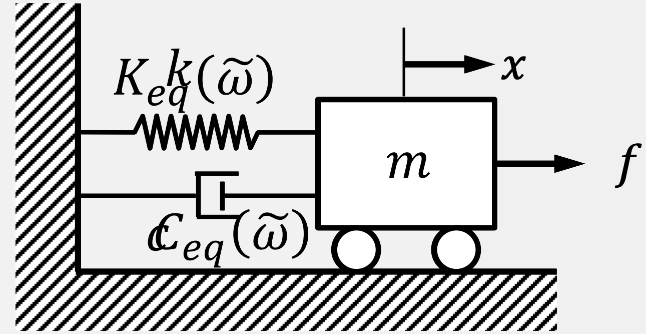
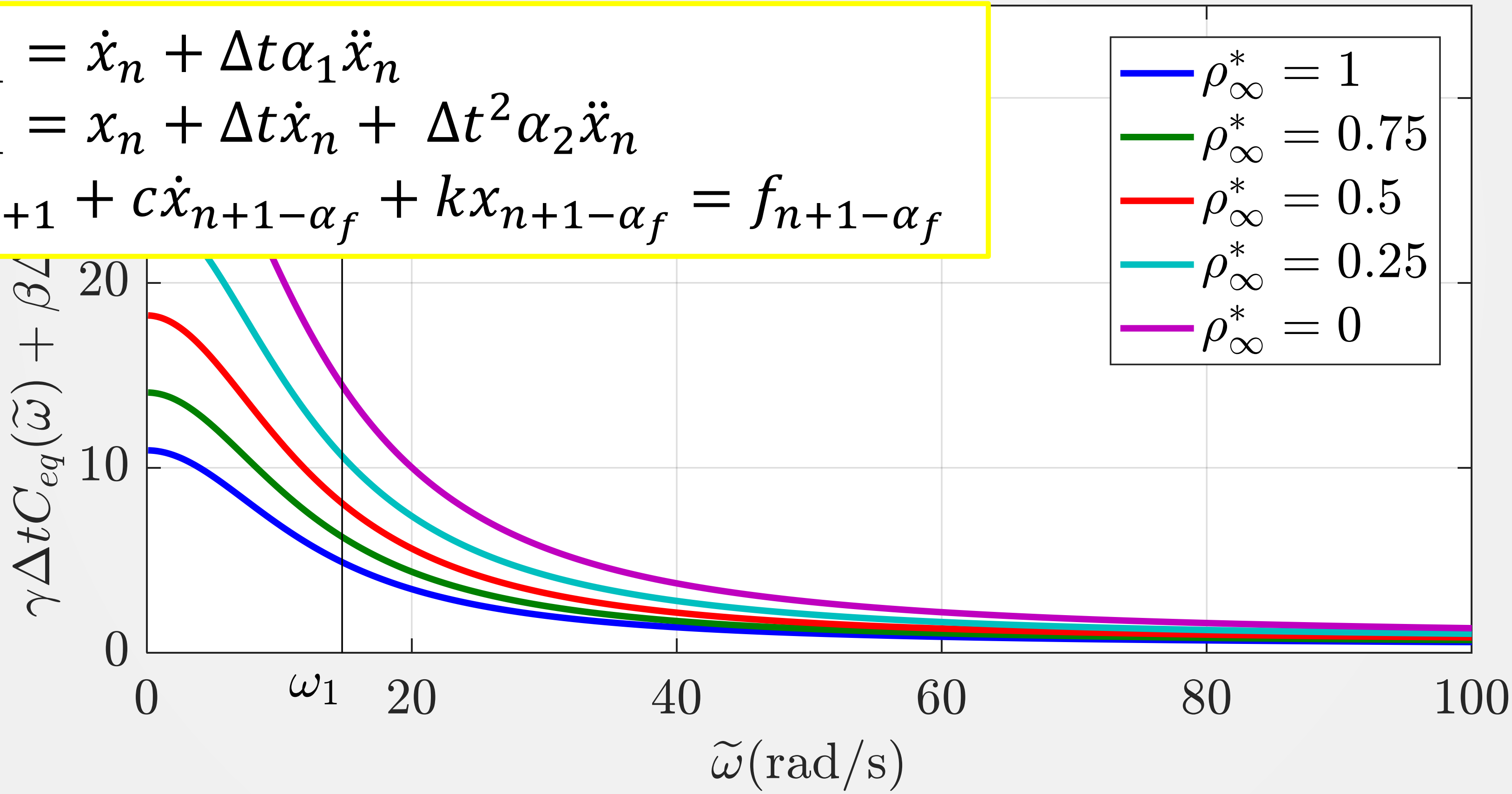
Influence of Model-Based Integration Parameters

$$\alpha_1 = \frac{m}{m+\boxed{\gamma\Delta t c+\beta\Delta t^2 k}};\ \alpha_2 = \left(\frac{1}{2}+\gamma\right)\alpha_1;$$

$$\alpha_3 = \frac{\alpha_m m+\alpha_f\gamma\Delta t c+\alpha_f\beta\Delta t^2 k}{m+\boxed{\gamma\Delta t c+\beta\Delta t^2 k}}$$

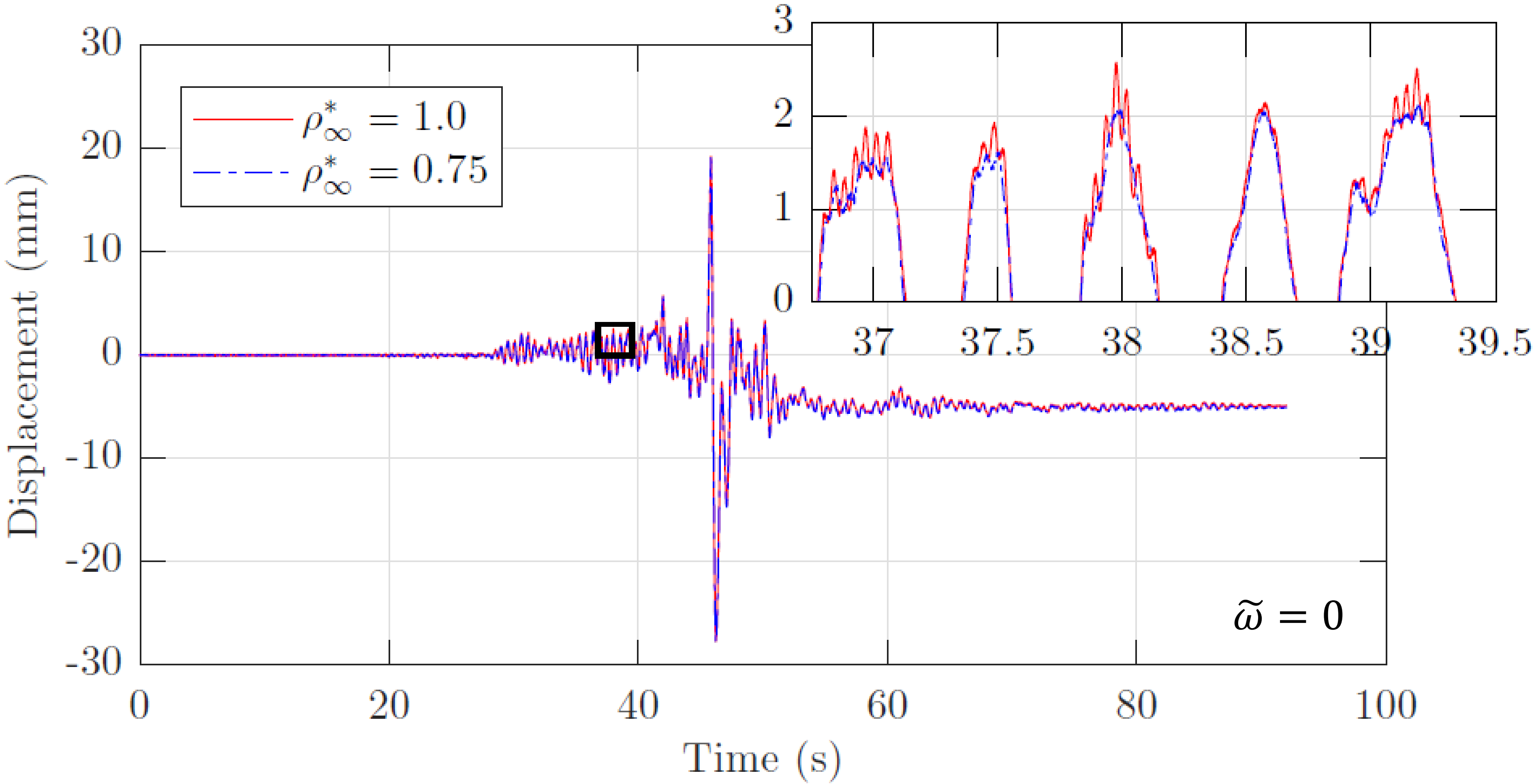


$$\dot{x}_{n+1} = \dot{x}_n + \Delta t\alpha_1\ddot{x}_n$$
$$x_{n+1} = x_n + \Delta t\dot{x}_n + \Delta t^2\alpha_2\ddot{x}_n$$
$$m\hat{\ddot{x}}_{n+1} + c\dot{x}_{n+1-\alpha_f} + kx_{n+1-\alpha_f} = f_{n+1-\alpha_f}$$

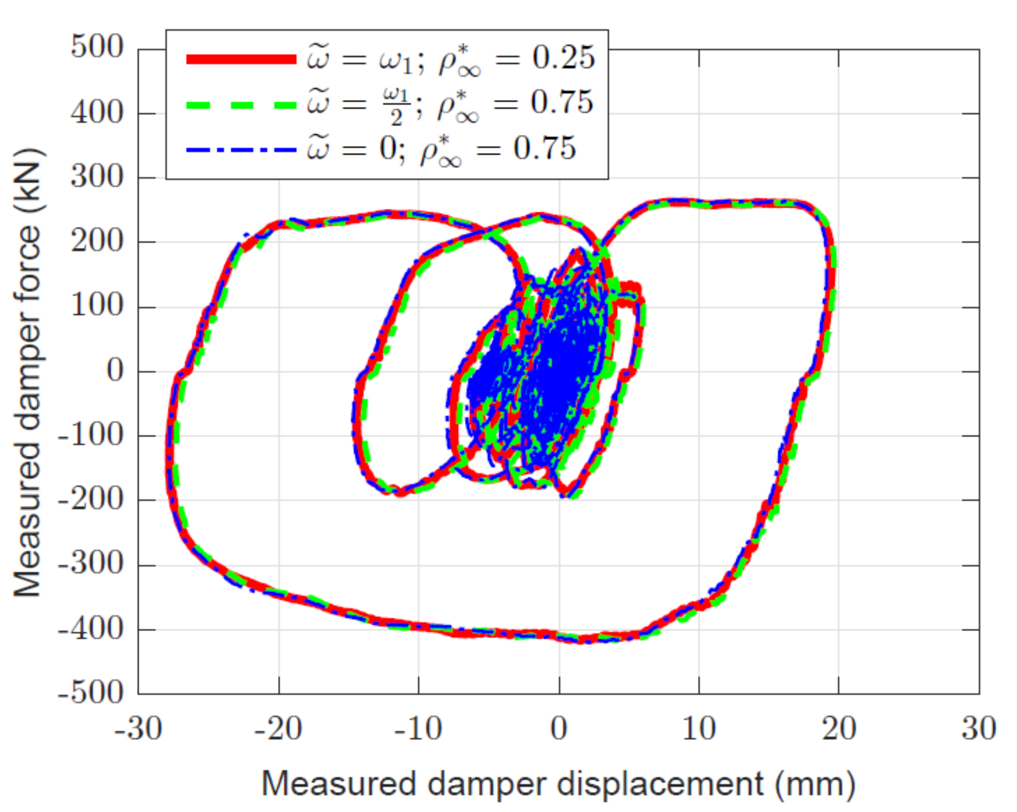


RTHS Test Matrix

- ❑ Influence of numerical dissipation and model-based parameters on stability and accuracy of RTHS results



Comparison of Selected RTHS: Accuracy



Comparison of story drifts (%)

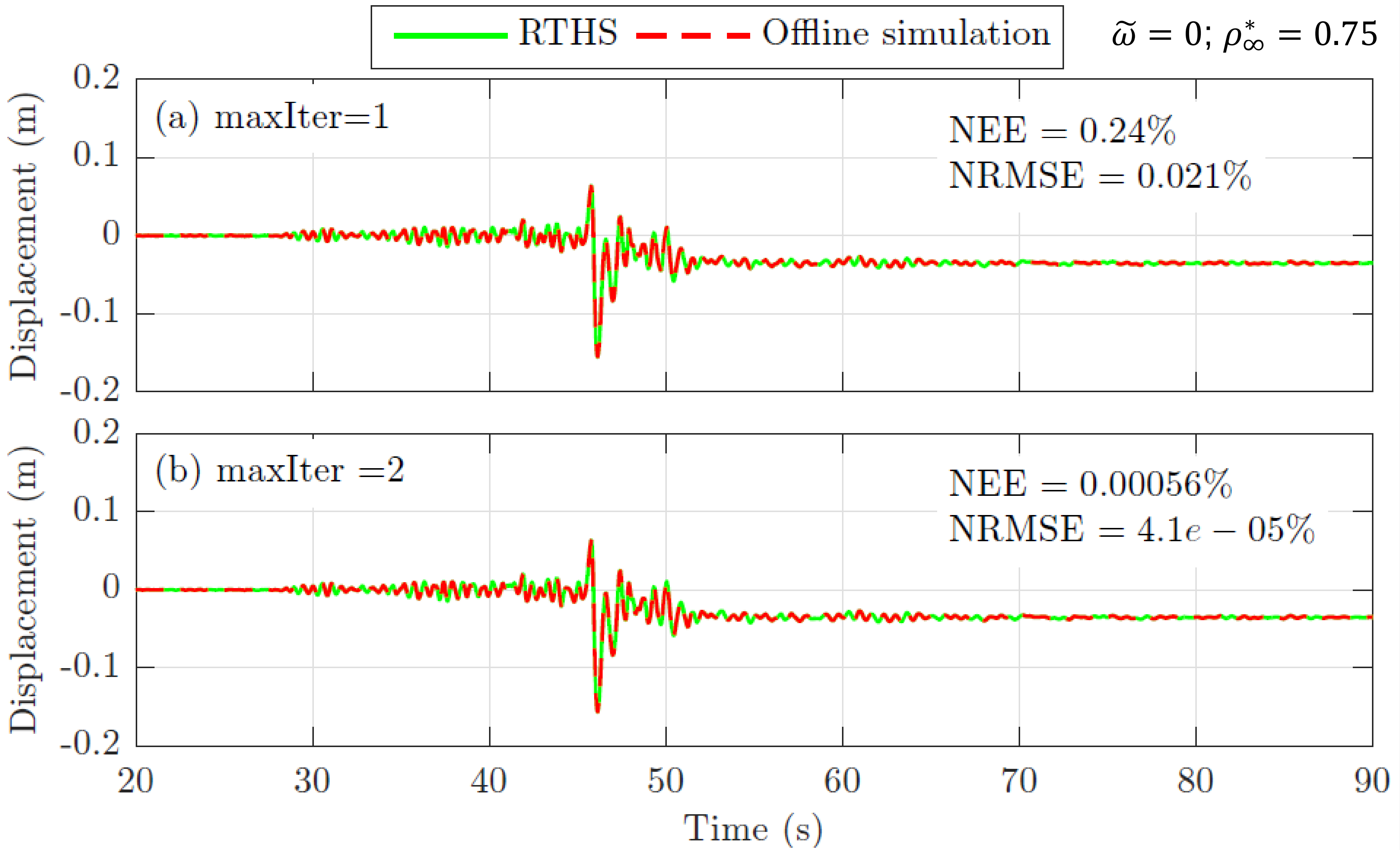
Story	$\widetilde{\omega} = \omega_1;$ $\rho^*_\infty = 0.25$	$\widetilde{\omega} = \frac{\omega_1}{2};$ $\rho^*_\infty = 0.75$	$\widetilde{\omega} = 0;$ $\rho^*_\infty = 0.75$
1	3.372	3.372	3.391
2	1.004	0.988	1.005

Accuracy is not influenced by $\widetilde{\omega}$ and ρ^*_∞, provided stability is achieved

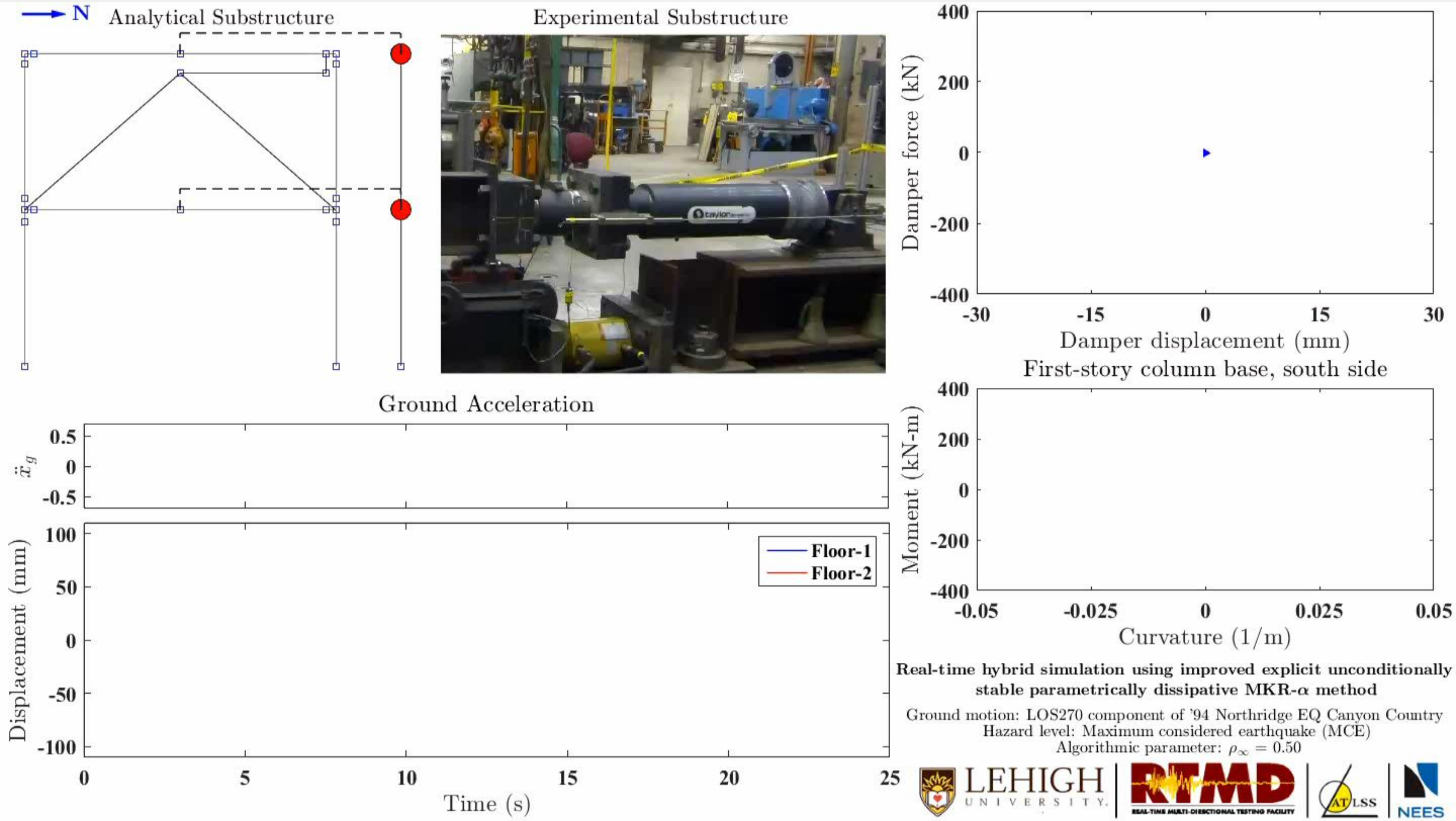
RTHS: Influence of Fixed Number of Iterations

- Numerical simulation of RTHS was performed (offline simulation)
- All FB elements were allowed to converge with $Etol = 10^{-16}$
- Required 8 iterations for most of the elements
- Measured damper force from the RTHS was used

RTHS: Influence of Fixed Number of Iterations



MCE Level Test Demonstration



Summary and Conclusions

- Direct integration algorithm
  - Reviewed the MKR-α method
  - Influence of model-based integration parameters on stability and accuracy of RTHS
  - Accuracy is not influenced by model-based integration parameters and numerical damping, provided stability is achieved
  - Controllable numerical energy dissipation in MKR-α method makes it well suited for RTHS of complex structures

- FE modeling of analytical substructure
  - Proposed an efficient implementation procedure for force-based elements for application to RTHS
  - Assessed the implementation using numerical and RTHS results
  - Proposed implementation procedure is well suited for RTHS and large-scale numerical simulations using explicit algorithms

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