Real-Time Hybrid Simulation of a Reinforced Concrete Building using Force-Based Elements and Advanced Explicit Integration Algorithms

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Outline

- Introduction
- Advanced explicit direct integration algorithms with numerical damping
  - Formulation
  - Numerical characteristics
- Force-based fiber element implementation
- Prototype structure
- Numerical assessment of element implementation scheme
- Real-time hybrid simulation (RTHS)
  - Model-based integration parameters
  - Stability, accuracy, and numerical dissipation
  - Influence of fixed number of element iterations
- Summary and conclusions
Introduction: RTHS

Integration of equations of motion

\[ M\ddot{X}_{n+1} + C\dot{X}_{n+1} + R_{n+1}^a + R_{n+1}^e = F_{n+1} \]

- **Integration algorithm**
- **FE modeling**

ATS Compensator (Chae et al., 2013)

Introduction: RTHS

Direct Integration Algorithms

- Explicit formulation
- Unconditional stability
- Controllable numerical damping
- Improved overshoot for high-frequency modes
- Improved stability for nonlinear stiffening type systems

FE Modeling of Analytical Substructure

**Displacement-based fiber elements**

- Curvature varies linearly
  - KR-\(\alpha\) method
    - Requires many elements per structural member to model nonlinear response
    - Increases number of DOFs
- State determination is straightforward

**Force-based fiber elements**

- Equilibrium is strictly enforced
  - Material nonlinearity can be modeled using a single element per structural member
  - Reduces number of DOFs
- Requires iterations at the element level

Modifed KR-\(\alpha\) method

Force-based fiber element implementation with fixed number of iterations
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Explicit Modified KR-\(\alpha\) (MKR-\(\alpha\)) Method

Velocity update:
\[
\dot{X}_{n+1} = \dot{X}_n + \Delta t \alpha_1 \ddot{X}_n
\]

Displacement update:
\[
X_{n+1} = X_n + \Delta t \dot{X}_n + \Delta t^2 \alpha_2 \ddot{X}_n
\]

Weighted equations of motion:
\[
M \ddot{X}_{n+1} + C \dot{X}_{n+1-\alpha_f} + KX_{n+1-\alpha_f} = F_{n+1-\alpha_f}
\]

where,
\[
\ddot{X}_{n+1} = (I - \alpha_3) \ddot{X}_{n+1} + \alpha_3 \ddot{X}_n
\]
\[
\dot{X}_{n+1-\alpha_f} = (1 - \alpha_f) \dot{X}_{n+1} + \alpha_f \dot{X}_n
\]
\[
X_{n+1-\alpha_f} = (1 - \alpha_f) X_{n+1} + \alpha_f X_n
\]
\[
F_{n+1-\alpha_f} = (1 - \alpha_f) F_{n+1} + \alpha_f F_n
\]

Initial acceleration:
\[
M \ddot{X}_0 = [F_0 - C \dot{X}_0 - KX_0]
\]

Integration Parameters

- Parameter controlling numerical energy dissipation
  - $\rho_\infty = \text{spectral radius when } \Omega = \omega \Delta t \to \infty$
  - varies in the range $0 \leq \rho_\infty \leq 1$
  - $\rho_\infty = 1$: No numerical energy dissipation
  - $\rho_\infty = 0$: Asymptotic annihilation

- Scalar integration parameters:
  - $\alpha_m = \frac{1}{\rho_\infty}$
  - $\alpha_f = \frac{1}{2} \left( \frac{1}{2} + \gamma \right)$
  - $\beta = \frac{1}{2} \left( \frac{1}{2} + \gamma \right)$

- Model-based integration parameter matrices:
  - $\alpha_1 = \left[ M_{IP} + \gamma \Delta t C_{IP} + \beta \Delta t^2 K_{IP} \right]^{-1} M_{IP}$;
  - $\alpha_2 = \left( \frac{1}{2} + \gamma \right) \alpha_1$
  - $\alpha_3 = \left[ M_{IP} + \gamma \Delta t C_{IP} + \beta \Delta t^2 K_{IP} \right]^{-1} \left[ \alpha_m M_{IP} + \alpha_f \gamma \Delta t C_{IP} + \alpha_f \beta \Delta t^2 K_{IP} \right]$

- $M_{IP}$, $C_{IP}$, and $K_{IP}$ need to be formed based on the hybrid system

Numerical Characteristics

Compare based on same high-frequency dissipation

\[ \rho_\infty^* = \begin{cases} 
\rho_\infty & \text{for KR-\(\alpha\) and G-\(\alpha\) methods} \\
\rho_\infty^2 & \text{for MKR-\(\alpha\) method}
\end{cases} \]

G-\(\alpha\): Implicit generalized-\(\alpha\) method
(Chung & Hulbert, 1993)

Dashed lines (- -): KR-\(\alpha\) and G-\(\alpha\); Solid lines (—): MKR-\(\alpha\)

\[ \Delta t = \text{integration time step size}; \ T = \text{undamped natural period of an SDOF oscillator} \]
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Force-Based (FB) Element State Determination

- Given element deformations $\mathbf{v}$, need element restoring forces $\mathbf{s}$
- Know the force interpolation function
  - Constant axial force and linear bending moment if no element loads
- State determination is not straightforward in a standard stiffness-based FE program

- Spacone et al. (1996) developed an iterative procedure
  - Not well suited for RTHS
- Neuenhofer and Filippou (1997) proposed a noniterative procedure
  - Uses iteration at the structure level (Newton-Raphson type)
  - Not applicable for RTHS using explicit algorithms

- New implementation scheme based on Spacone et al. (1996) and Neuenhofer and Filippou (1997)
  - Fixed number of iterations
  - Carry over unbalanced section forces and correct in the next time step
Element Convergence Criteria

- Employed the energy based criteria (Taucer et al., 1991)

\[
(NEI^j)_{n+1} = \frac{(EI^j)_{n+1}}{(EI^{j=1})_{n+1}} \leq Etol \quad \text{for } j > 1
\]

where

\[
(EI^j)_{n+1} = (\Delta s^j)^T (\Delta v^j) = (\Delta v_{r}^{j-1})^T K^{-1} (\Delta v_{r}^{j-1})
\]

\[
(EI^{j=1})_{n+1} = (\Delta s^{j=1})^T (\Delta v^{j=1}) = ((\Delta v)_{n+1})^T (K)_{n} (\Delta v)_{n+1}
\]

- A typical value of \( Etol = 10^{-16} \) is used (Taucer et al., 1991)
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Prototype and RTHS Configuration

- Two-story RC SMRF with nonlinear viscous dampers
- Retail store located in Los Angeles area on a stiff soil site

1999 Chi-Chi EQ record scaled to MCE hazard level
Time step: $\Delta t = \frac{3}{1024}$ s

Prototype floor plan

Plastic hinge integration (Scott and Fenves, 2006)
Modeling of Inherent Damping

- In RTHS using explicit algorithms generally mass and initial stiffness proportional damping is used
  - Known to produce unrealistically large damping forces when structure undergoes significant inelastic deformations
  - Can use nonproportional damping (Kolay et al., 2015)
    - Not a good model for FB elements because deformations localize at some integration points not in an entire element

- Use tangent stiffness for FB elements; it is readily available
  - For other elements, if any, use initial stiffness
  - Damping forces are calculated for each FB element inside state determination process
  - 3% damping to first \((T_1 = 0.43 \text{ s})\) and second modes \((T_2 = 0.12 \text{ s})\) of system

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Assessment of FB Element Implementation

- Consider only the RC SMRF
- Perform numerical simulation using the same ground motion
- Study the influence of max number of iterations ($maxIter$) with CO=Yes and CO= No based on a comparison with a reference solution
  - Reference solution: Newmark average acceleration algorithm and all the FB elements were allowed to converge with $Etol = 10^{-16}$
- Numerical damping is not required: $\rho_\infty^* = 1.0$
- Time step $\Delta t = \frac{3}{1024} \text{ s}$, smallest time step that can be used in real-time for the RTHS configuration with $maxIter = 2$ for all FB elements
Assessment of FB Element Implementation

Roof displacement from numerical simulation

- CO=No with $maxIter = 1$ produces large error
- CO=Yes compare well with reference
- Even $maxIter = 1$ with CO produces acceptable results
- Increasing $maxIter$ increases accuracy

\[
\text{NEE} = \frac{\sum_{n=1}^{N} (x_n)^2 - \sum_{n=1}^{N} (\bar{x}_n)^2}{\sum_{n=1}^{N} (\bar{x}_n)^2}
\]

\[
\text{NRMSE} = \frac{\sqrt{\frac{1}{N} \sum_{n=1}^{N} (x_n - \bar{x}_n)^2}}{\max(\bar{x}) - \min(\bar{x})}
\]
Assessment of FB Element Implementation

Moment-curvature response from numerical simulation at the first-story south side column base
Assessment of FB Element Implementation

Energy increment \( (EI_{j=\text{maxIter}+1})_{n} = (\Delta s_{j})^T (\Delta v_{j}) \) for first-story south side column element from numerical simulation
Assessment of FB Element Implementation

Peak story-drift (%) from numerical simulations with CO=Yes

<table>
<thead>
<tr>
<th>Story</th>
<th>Reference</th>
<th>$maxIter = 1$</th>
<th>$maxIter = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.566</td>
<td>2.536</td>
<td>2.547</td>
</tr>
<tr>
<td>2</td>
<td>2.925</td>
<td>2.913</td>
<td>2.906</td>
</tr>
</tbody>
</table>
Assessment of FB Element Implementation

- CO=Yes produces an accurate result even if no iteration is performed at the element level \((maxIter = 1)\)
- Benefit of CO=Yes reduces with increasing \(maxIter\)
- It is useful to perform the carry over (CO=Yes) because additional computation effort is small
  - Use only CO=Yes for RTHS
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RTHS: Model-Based Integration Parameters

- Model-based integration parameters ($\alpha_1$, $\alpha_2$, and $\alpha_3$) require $M_{IP}$, $C_{IP}$, and $K_{IP}$

- For the present study
  - $M_{IP} = M =$ analytically modeled mass matrix
    - Experimental substructure mass is small
  - $C_{IP} = (a_0 M + a_1 K^a_I) + C^e_{eq}$
    - $K^a_I =$ initial stiffness matrix of analytical substructure
    - $C^a_{eq} =$ equivalent damping matrix of experimental substructure
    - $a_0$ and $a_1$ are Rayleigh damping coefficients
  - $K_{IP} = K^a_I + K^e_{eq}$
    - $K^e_{eq} =$ equivalent stiffness matrix of experimental substructure

- How can we determine $C^e_{eq}$ and $K^e_{eq}$?
Damper Characterization

Model parameters identified using particle swarm optimization algorithm (PSO)

\[ K_D = 9.49 \times 10^4 \text{kN/m}, \]
\[ C_D = 644.96 \text{kN/(s/m)} \]
\[ \alpha = 0.439 \]

Nonlinear Maxwell damper model

\[ u_D, f_D \]
\[ u_K, f_K \]
\[ u_C, f_C \]
Model-Based Integration Parameters

Linearization of nonlinear Maxwell model at a small velocity (0.5 in/s) and determination of frequency dependent equivalent Kelvin-Voigt model parameters

What is the value of $\tilde{\omega}$?
RTHS Results: Instability!

\[ \rho^*_\infty = 0.50, \ \tilde{\omega} = \omega_1 \text{ and } maxIter = 2 \text{ for all FB elements} \]

### High-frequency oscillations:

- **Causes:**
  - Underestimation error in \( C_{eq} \) & \( K_{eq} \)
  - Noise in restoring forces
  - ATS compensator amplifying higher frequencies

- **Remedies:**
  - Add more numerical damping
  - Increase \( C_{eq} \) and \( K_{eq} \)

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Influence of Model-Based Integration Parameters

\[ \alpha_1 = \frac{m}{m + \gamma \Delta t c + \beta \Delta t^2 k}; \quad \alpha_2 = \left( \frac{1}{2} + \gamma \right) \alpha_1; \]
\[ \alpha_3 = \frac{\alpha_m m + \alpha_f \gamma \Delta t c + \alpha_f \beta \Delta t^2 k}{m + \gamma \Delta t c + \beta \Delta t^2 k} \]

\[ \dot{x}_{n+1} = \dot{x}_n + \Delta t \alpha_1 \ddot{x}_n \]
\[ x_{n+1} = x_n + \Delta t \dot{x}_n + \Delta t^2 \alpha_2 \ddot{x}_n \]
\[ m \ddot{x}_{n+1} + c \dot{x}_{n+1} - \alpha_f + k x_{n+1} - \alpha_f = f_{n+1} - \alpha_f \]
RTHS Test Matrix

- Influence of numerical dissipation and model-based parameters on stability and accuracy of RTHS results

\[ \omega_1 \sim \text{Stable} \quad (10.64) \]

\[ \omega_2 \sim \text{Stable} \quad (18.24) \]

\[ \rho_\infty^* = 1.0 \]

\[ \rho_\infty^* = 0.75 \]

\[ \tilde{\omega} = 0 \]
Comparison of Selected RTHS: Accuracy

Comparison of story drifts (%)

<table>
<thead>
<tr>
<th>Story</th>
<th>$\tilde{\omega} = \omega_1; \rho_\infty^* = 0.25$</th>
<th>$\tilde{\omega} = \frac{\omega_1}{2}; \rho_\infty^* = 0.75$</th>
<th>$\tilde{\omega} = 0; \rho_\infty^* = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.372</td>
<td>3.372</td>
<td>3.391</td>
</tr>
<tr>
<td>2</td>
<td>1.004</td>
<td>0.988</td>
<td>1.005</td>
</tr>
</tbody>
</table>

Accuracy is not influenced by $\tilde{\omega}$ and $\rho_\infty^*$, provided stability is achieved.
RTHS: Influence of Fixed Number of Iterations

- Numerical simulation of RTHS was performed (offline simulation)
- All FB elements were allowed to converge with $E_{tol} = 10^{-16}$
- Required 8 iterations for most of the elements
- Measured damper force from the RTHS was used
RTHS: Influence of Fixed Number of Iterations

\[ \tilde{\omega} = 0; \quad \rho^* = 0.75 \]

(a) maxIter = 1

NEE = 0.24%
NRMSE = 0.021%

(b) maxIter = 2

NEE = 0.00056%
NRMSE = 4.1e-05%
MCE Level Test Demonstration

Analytical Substructure

Experimental Substructure

Ground Acceleration

Real-time hybrid simulation using improved explicit unconditionally stable parametrically dissipative MKR-α method

Ground motion: LOS270 component of '94 Northridge EQ Canyon Country Hazard level: Maximum considered earthquake (MCE)

Algorithmic parameter: $\rho_\infty = 0.50$

First-story column base, south side

Damper force (kN)

Damper displacement (mm)

Curvature (1/m)

Displacement (mm)

Moment (kN-m)

Time (s)
Summary and Conclusions

- Direct integration algorithm
  - Reviewed the MKR-\(\alpha\) method
  - Influence of model-based integration parameters on stability and accuracy of RTHS
  - Accuracy is not influenced by model-based integration parameters and numerical damping, provided stability is achieved
  - Controllable numerical energy dissipation in MKR-\(\alpha\) method makes it well suited for RTHS of complex structures

- FE modeling of analytical substructure
  - Proposed an efficient implementation procedure for force-based elements for application to RTHS
  - Assessed the implementation using numerical and RTHS results
  - Proposed implementation procedure is well suited for RTHS and large-scale numerical simulations using explicit algorithms
References


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Thank you